PQ 7 Heat

Q and A

Data for Q1 to Q3

Specific heat capacity of water = 4200 $J\ kg^{-1}\ K^{-1}$.

Specific heat capacity of iron at 100 °C = 220 $J\ kg^{-1}\ K^{-1}$.

Latent heat of vaporisation of water = 2260 $kJ\ kg^{-1}$.

Latent heat of fusion of water = 334 $kJ\ kg^{-1}$.

Density of ice at 0°C is 917 $kg\ m^{-3}$.

Q1

- Compare the energy needed to raise the temperature of 1kg of water from 20°C to 100°C and the energy needed to boil 1 kg of water at 100°C.

Energy required to heat the water = $mC\Delta\theta = 1 \times 4200 \times 80 = 336$ kJ

Energy required to boil the water = $mL = 1 \times 2260$ kJ = 2260kJ.

So it takes nearly 7 times as much energy to boil the water as to heat it up.

Q2

A 5kg block of iron is heated to 800°C. It is placed in a tub containing 2 litre of water at 15°C. Assuming all the water is brought to the boil rapidly; calculate the mass of water which boils off.

Energy given up by the iron in cooling to 100°C (assuming c is constant)

$= mC\Delta\theta = 5 \times 700 \times 220 = 770$ kJ.

To heat 2 litre of water from 15°C to 100°C requires $2 \times 4200 \times (100-15) = 714$ kJ.

This leaves $770 - 714 = 56$ kJ to boil some of the water.

Mass of water boiled away = $E/L = 56 / 2260 = 0.025$ kg.

Q3

Sunlight of intensity 0.6 kW m^{-2} falls on a patch of ice. Assuming the ice absorbs 20% of the light; calculate what thickness of ice would melt in 1 minute. (Assume any water produced runs off).

In 1 minute $600 \times 60 \times 0.50 = 18000$ J are absorbed by the ice for each 1 m^2. Let the ice have an area A. The energy absorbed is 18000 A.

Thickness t will have volume At and will be melted if mass $\times$ latent heat of fusion = 18000 A.

The mass of volume At ice is 917At.

So 18000 A = 917 At $\times$ 334000

$t = 0.6 \times 10^{-4}$ m or 0.06 mm

Q4

How much energy is required to change 2.6kg of **ice at 0˚C** into **water** at the same temperature?

$\mathbf{E_h = mL_f}$

$= 2.6 \times (3.34 \times 10^5)$

$E_h = \underline{8.7 \times 10^5\ J}$

E_h = heat energy (J)

m = mass = 2.6kg

L_f = 3.34×10^5 J/kg

Q5

How much energy is required to change 2.6kg of **water** at **100°C** into **steam** at the same temperature?

$$\mathbf{E_h = mL_v}$$

$$= 2.6 \times (2.26 \times 10^6 J)$$

$$E_h = \underline{5.9 \times 10^6\ J}$$

E_h = heat energy (J)
m = mass = 2.6kg
L_v = 2.26×10^6 J/kg

Q6

How much energy is required in total to change 1.9kg of ice at -10˚C to steam at 100˚C?

Split the problem into parts.

We need to calculate the energy required to

1. Heat the ice from -10˚C to 0˚C
2. melt all of the ice
3. heat the liquid water up to 100˚C
4. change the boiling water into steam.

then add all 4 answers together to get the total energy required.

Q6 continued

Energy to heat ice up to melting point:

$$E_h = cm\Delta T$$

$$= 2100 \times 1.9 \times 10$$

$$\underline{E_h = 39{,}900 \text{ J}}$$

E_h = heat energy (J)
m = mass = 1.9kg
ΔT = 10
c = specific heat capacity of **ice** (= 2100J/kg/°C)

Q6 continued

Energy to melt the ice:

$$\mathbf{E_h = mL_f}$$

where E_h = heat energy (J)
m = mass = 1.9kg
L_f = 3.34×10^5 J/kg

$$= 1.9 \times (3.34 \times 10^5)$$

E_h = 634,600 J

Q6 continued

Energy to heat the water up to boiling point:

$$\mathbf{E_h = cm\Delta T}$$

$$= 4200 \times 1.9 \times 100$$

$$\underline{E_h = 798{,}000\ J}$$

E_h = heat energy (J)
m = mass = 1.9kg
ΔT = 100
c = specific heat capacity of **water** (= 4200J/kg/°C)

Q6 continued

Energy to convert the boiling water into steam;

$\mathbf{E_h = mL_v}$

$= 1.9 \times (2.26 \times 10^6)$

where E_h = heat energy (J)

m = mass = 1.9kg

$L_v = 2.26 \times 10^6$ J/kg

$\underline{E_h = 4{,}294{,}000\ J}$

Q6 continued

Now add all 4 answers together to get the total energy required.

Energy required = 39,900J + 634,600J + 798,000J + 4,294,000J

= **5,766,500 Joules**

Q7

- The specific latent heat of fusion (melting) of ice is 334 000 J/kg. How much energy is needed to melt 5kg of ice at 0°C to 5 kg of water at 0°C?

- Energy = mL = 5 x 334 000 = 1670000 J

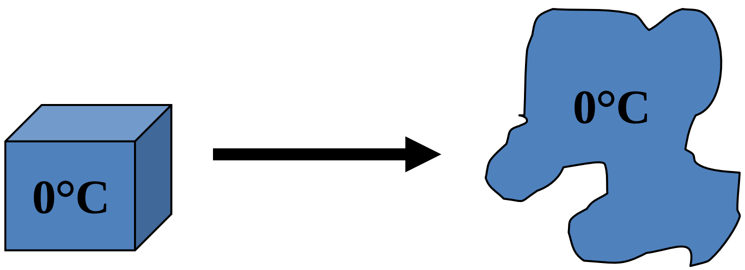
Q8

- Calculate the amount of heat required to completely convert 50 g of ice at 0 ºC to steam at 100 ºC. The specific heat capacity of water is 4.18 kJ/kg/°C. The specific latent heat of fusion of ice is 334 kJ/kg, and the specific heat of vaporization of water is 2260 kJ/kg.

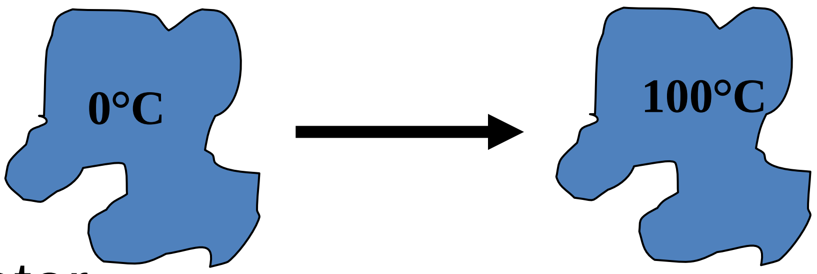
Q8 continued

Heat is taken up in three stages:

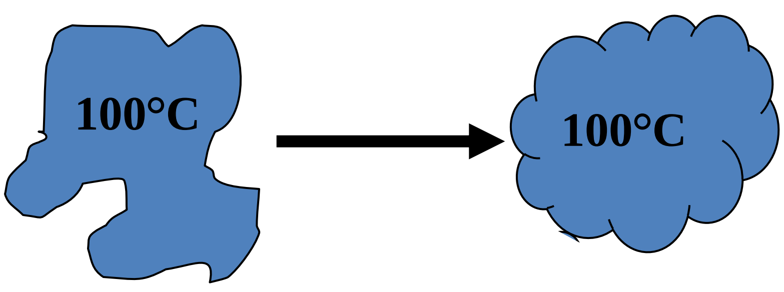
1. The melting of the ice.



2. The heating of the water.

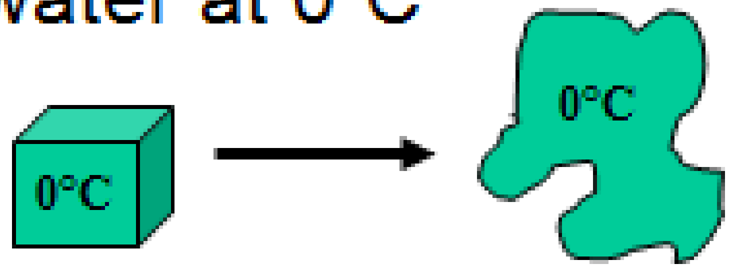


3. The vaporization of the water.



Stage 1

1. Heat taken up for converting ice at 0°C to water at 0°C



mass of water x latent heat of fusion

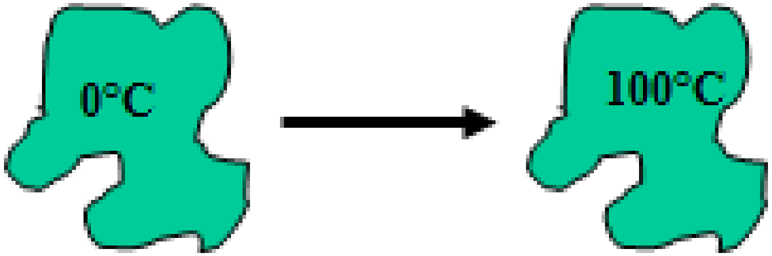
= 0.050 (kg) x 334 ($kJ.kg^{-1}$)

= 16.7 kJ

Q8 continued

Stage 2

2. Heat taken up heating the water from 0 °C to the boiling point, 100 °C



mass of water x specific heat capacity x temperature change

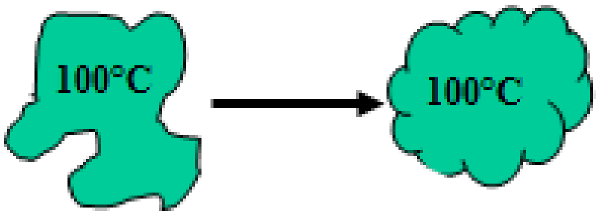
= 0.05 (kg) x 4.18 ($kJ.kg^{-1}.°C^{-1}$)x 100 (°C)

= 20.9 kJ

Q8 continued

Stage 3

3. Heat taken up vaporising the water



mass of water x latent heat of vaporization
$0.05\ (kg) \times 2260\ kJ.kg^{-1}$
$= 113\ kJ$

Q8 continued

The answer

The sum of these is

16.7 + 20.9 + 113
= 150.6 kJ (151 kJ)

Q9

In a cappuccino machine, steam is passed into milk to heat it to a final temperature of 85°C. Calculate the rise in temperature, $\Delta\theta$ of 0.2kg of milk when 1.0g of steam at a temperature of 100°C is passed into it. Assume that the thermal energy losses to the surroundings are negligible.

Specific heat capacity of milk = 4.0 kJ kg^{-1} K^{-1}

Specific heat capacity of water = 4.2 kJ kg^{-1} K^{-1}

Specific latent heat of vaporization of water= 2.2 MJ kg^{-1}

Heat gained by milk = Heat loss by steam

$$(0.2)(4000)(\Delta\theta) = (0.001)\left(2.2\times10^{6}\right) + (0.001)(4200)(15)$$

Rise in temperature of milk, $\Delta\theta$ = 2.83 K (ans)

Q10

A holiday park has an open-air swimming pool of circular design shown in the figure below. The mass of water in the pool is about 8.5×10^5 kg

The water in the pool is heated to 30°C. In July, the temperature of the water taken from the supply is 9°C. Calculate the energy needed to raise the temperature of all the water in the pool from 9 °C to 30 °C assuming there are no energy losses.

Specific heat capacity of water= 4200 J $kg^{-1}K^{-1}$

$$Q = mc\Delta\theta = 8.5 \times 10^5 \times 4200 \times 21 = 7.5 \times 10^{10} \text{ J}$$

Q11

- In order to extract the maximum flavour in the shortest amount of time, your local fast food purveyor has decided to brew its coffee at 90 °C and serve it quickly so that it has only cooled down to 85 °C. While this may be economically sensible, it is negligent and dangerous from a health and safety standpoint. Water (which is what coffee mostly is) at 85 °C is hot enough to cause third-degree burns (the worst kind) in two to seven seconds. You decide to add ice cubes to your coffee to cool it down to a more reasonable 55 °C so you will be able to drink it sooner. How many 23.5 g ice cubes at −18.5 °C should you add to your 355 ml cup of coffee to accomplish your thermal goal?

Q11 continued

- This is a conservation of energy problem. The heat gained by the ice will be equal to the heat lost by the coffee.

$$+Q_{ice} = -Q_{coffee}$$

- The ice must first warm up to its melting point (a temperature change), then it has to melt (a phase change), and then the liquid has to warm up (another temperature change). The coffee has less to do. It just has to cool down.

$$\begin{aligned} & Q_{cold\ ice} \\ & + Q_{melting} \\ & + Q_{melted\ ice} \\ \hline & = -Q_{coffee} \end{aligned} \qquad \begin{aligned} & [mc\Delta T]_{cold\ ice} \\ & + [mL]_{melting} \\ & + [mc\Delta T]_{melted\ ice} \\ \hline & = -[mc\Delta T]_{coffee} \end{aligned}$$

Q11 continued

- The final mixture will end up at one temperature.

$$\frac{m_{ice}(2{,}090\ \text{J/kg C°})(0 - -18.5\ \text{°C}) + m_{ice}(334{,}000\ \text{J/kg}) + m_{ice}(4{,}200\ \text{J/kg C°})(55 - 0\ \text{°C})}{= -(0.355\ \text{kg})(4{,}200\ \text{J/kg C°})(55 - 85\ \text{°C})}$$

$$m_{ice}(603{,}665\ \text{J/kg}) = (44{,}730\ \text{J})$$

$$m_{ice} = 0.0741\ \text{kg}$$

- This is about three ice cubes.

  $number = (74.1\ \text{g})/(23.5\ \text{g}) \approx 3$ ice cubes

Q12

A small object of mass $m = 0.26\,\text{kg}$ and unknown specific heat c, initially at 100 °C, is placed into a body of liquid with a heat capacity of $1300\,\text{J}\,\text{K}^{-1}$, initially at 20 °C. The final equilibrium temperature is 25 °C. What is the value of c? Ignore any heat that may be absorbed by the vessel containing the liquid.

Solution To reach thermal equilibrium, the object cools by 75 K, and the liquid is warmed by 5 K, so (assuming there are no heat losses)

$$\text{heat gained by liquid} = 1300\,\text{J}\,\text{K}^{-1} \times 5\,\text{K} = 6500\,\text{J}$$

$$\text{heat lost by object} = mc \times 75\,\text{K}$$

As these two values are equal, we can write

$$mc = 6500\,\text{J}/75\,\text{K} = 86.7\,\text{J}\,\text{K}^{-1}$$

Thus, $c = 86.7\,\text{J}\,\text{K}^{-1}/0.26\,\text{kg} = 333\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$