Worked Examples

Heat

Worked Example 1

- A 0.500 kg aluminum pan on a stove is used to heat 0.250 liters of water from 20.0ºC to 80ºC . (a) How much heat is required? What percentage of the heat is used to raise the temperature of (b) the pan and (c) the water?

Solids	J/kg·ºC
Aluminum	900

- The pan and the water are always at the same temperature. When you put the pan on the stove, the temperature of the water and the pan is increased by the same amount. We use the equation for the heat transfer for the given temperature change and mass of water and aluminum.
- Calculate the temperature difference:

$$\Delta T = T_f - T_i = 60.0^\circ\text{C}.$$

Worked Example 1

- Calculate the mass of water.
- The density of water is 1000 kg/m3 , one litre of water has a mass of 1 kg, and the mass of 0.250 litres of water is

$$m_{\mathrm{W}} = 0.250\ \mathrm{kg}$$

- Calculate the heat transferred to the water.

$$Q_{\mathrm{W}} = m_{\mathrm{W}} c_{\mathrm{W}} \Delta T = (0.250\ \mathrm{kg})(4186\ \mathrm{J/kg^{\circ}C})(60.0^{\circ}\mathrm{C}) = 62.8\ \mathrm{kJ}.$$

- Calculate the heat transferred to the aluminum

$$Q_{\mathrm{Al}} = m_{\mathrm{Al}} c_{\mathrm{Al}} \Delta T = (0.500\ \mathrm{kg})(900\ \mathrm{J/kg^{\circ}C})(60.0^{\circ}\mathrm{C}) = 27.0 \times 10^{4}\ \mathrm{J} = 27.0\ \mathrm{kJ}$$

Worked Example 1 continued

- Compare the percentage of heat going into the pan versus that going into the water.

$$Q_{\text{Total}} = Q_{\text{W}} + Q_{\text{Al}} = 62.8\ \text{kJ} + 27.0\ \ \text{kJ} = 89.8\ \text{kJ}.$$

- Thus, the amount of heat going into heating the pan is

$$\frac{27.0\ \text{kJ}}{89.8\ \text{kJ}} \times 100\% = 30.1\%.$$

- and the amount going into heating the water is

$$\frac{62.8\ \text{kJ}}{89.8\ \text{kJ}} \times 100\% = 69.9\%.$$

Note

- In this example, the heat transferred to the container is a significant fraction of the total transferred heat.
- Although the mass of the pan is twice that of the water, the specific heat of water is over four times greater than that of aluminum.
- Therefore, it takes a bit more than twice the heat to achieve the given temperature change for the water as compared to the aluminum pan.

Worked Example 2

- Lorry brakes used to control speed on a downhill run do work, converting gravitational potential energy into increased internal energy (higher temperature) of the brake material.
- This conversion prevents the gravitational potential energy from being converted into kinetic energy of the lorry.
- The problem is that the mass of the lorry is large compared with that of the brake material absorbing the energy, and the temperature increase may occur too fast for sufficient heat to transfer from the brakes to the environment.

- Calculate the temperature increase of 100 kg of brake material with an average specific heat of 800 J/kg · ºC if the material retains 10% of the energy from a 10,000-kg lorry descending 75.0 m (in vertical displacement) at a constant speed.

Worked Example 2

- Recall that If the brakes are not applied, gravitational potential energy is converted into kinetic energy.
- When brakes are applied, gravitational potential energy is converted into internal energy of the brake material.
- We first calculate the gravitational potential energy (Mgh) that the entire lorry loses in its descent and then find the temperature increase produced in the brake material alone.
- Calculate the change in gravitational potential energy as the lorry goes downhill

$$Mgh = (10{,}000\ \text{kg})\left(9.80\ \text{m/s}^2\right)(75.0\ \text{m}) = 7.35\times10^6\ \text{J}$$

Worked Example 2 continued

- Calculate the temperature from the heat transferred using Q=Mgh and

$$Q = Mgh \text{ and}$$

$$\Delta T = \frac{Q}{mc},$$

- Insert the values m = 100 kg and c = 800 J/kg · ºC to find

$$\Delta T = \frac{(7.35 \times 10^6 \text{ J})}{(100 \text{ kg})(800 \text{ J/kg°C})} = 92\text{°C}.$$

Note

- This temperature is close to the boiling point of water.
- If the lorry had been traveling for some time, then just before the descent, the brake temperature would likely be higher than the ambient temperature.
- The temperature increase in the descent would likely raise the temperature of the brake material above the boiling point of water, so this technique is not practical.

Worked Example 3

- Pour 0.250 kg of 20.0ºC water (about a cup) into a 0.500-kg aluminum pan off the stove with a temperature of 150ºC .
- Assume that the pan is placed on an insulated pad and that a negligible amount of water boils off.
- What is the temperature when the water and pan reach thermal equilibrium a short time later?

Worked Example 3 continued

- The pan is placed on an insulated pad so that little heat transfer occurs with the surroundings.
- Originally the pan and water are not in thermal equilibrium: the pan is at a higher temperature than the water.
- Heat transfer then restores thermal equilibrium once the water and pan are in contact.
- Because heat transfer between the pan and water takes place rapidly, the mass of evaporated water is negligible and the magnitude of the heat lost by the pan is equal to the heat gained by the water.
- The exchange of heat stops once a thermal equilibrium between the pan and the water is achieved.
- The heat exchange can be written as

$$Q_{\text{hot}} = Q_{\text{cold}}$$

Worked Example 3 continued

- Use the equation for heat transfer Q = mcΔT to express the heat lost by the aluminum pan in terms of the mass of the pan, the specific heat of aluminum, the initial temperature of the pan, and the final temperature:

$$Q_{\text{hot}} = m_{\text{Al}} c_{\text{Al}} (T_{\text{f}} - 150°\text{C}).$$

- Express the heat gained by the water in terms of the mass of the water, the specific heat of water, the initial temperature of the water and the final temperature:

$$Q_{\text{cold}} = m_{\text{W}} c_{\text{W}} (T_{\text{f}} - 20.0°\text{C}).$$

Worked Example 3 continued

- Note that Qhot < 0 and Qcold > 0 and that they must sum to zero because the heat lost by the hot pan must be the same as the heat gained by the cold water:

$$
\begin{aligned}
Q_{\text{cold}} + Q_{\text{hot}} &= 0, \\
Q_{\text{cold}} &= -Q_{\text{hot}}, \\
m_{\text{W}} c_{\text{W}} (T_{\text{f}} - 20.0°\text{C}) &= -m_{\text{Al}} c_{\text{Al}} (T_{\text{f}} - 150°\text{C}.)
\end{aligned}
$$

Worked Example 3 continued

- Bring all terms involving Tf on the left hand side and all other terms on the right hand side. Solve for Tf ,

$$T_f = \frac{m_{Al} c_{Al} (150°C) + m_W c_W (20.0°C)}{m_{Al} c_{Al} + m_W c_W}$$

$$T_f = \frac{(0.500\ kg)(900\ J/kg°C)(150°C) + (0.250\ kg)(4186\ J/kg°C)(20.0°C)}{(0.500\ kg)(900\ J/kg°C) + (0.250\ kg)(4186\ J/kg°C)}$$

$$= \frac{88430\ J}{1496.5\ J/°C}$$

$$= 59.1°C$$

Note

- This is a typical calorimetry problem—two bodies at different temperatures are brought in contact with each other and exchange heat until a common temperature is reached.
- Why is the final temperature so much closer to 20.0ºC than 150ºC ?
- The reason is that water has a greater specific heat than most common substances and thus undergoes a small temperature change for a given heat transfer.
- A large body of water, such as a lake, requires a large amount of heat to increase its temperature appreciably.
- This explains why the temperature of a lake stays relatively constant during a day even when the temperature change of the air is large.
- However, the water temperature does change over longer times (e.g., summer to winter).

Question?

- If 25 kJ is necessary to raise the temperature of a block from 25ºC to 30ºC , how much heat is necessary to heat the block from 45ºC to 50ºC ?

- The heat transfer depends only on the temperature difference. Since the temperature differences are the same in both cases, the same 25 kJ is necessary in the second case.

Worked Example 4

- Three ice cubes are used to chill a soda at 20ºC with mass m_{soda} = 0.25 kg .
- The ice is at 0ºC and each ice cube has a mass of 6.0 g.
- Assume that the soda is kept in a foam container so that heat loss can be ignored.
- Assume the soda has the same heat capacity as water.
- Find the final temperature when all ice has melted.

Worked Example 4 continued

- The ice cubes are at the melting temperature of 0ºC . Heat is transferred from the soda to the ice for melting.
- Melting of ice occurs in two steps:
- First the phase change occurs and solid (ice) transforms into liquid water at the melting temperature, then the temperature of this water rises.
- Melting yields water at 0ºC , so more heat is transferred from the soda to this water until the water plus soda system reaches thermal equilibrium,

$$Q_{\text{ice}} = -Q_{\text{soda}}$$

Worked Example 4 continued

- The heat transferred to the ice is

$$Q_{\text{ice}} = m_{\text{ice}} L_{\text{f}} + m_{\text{ice}} c_{\text{W}} (T_{\text{f}} - 0^{\circ}\text{C})$$

- The heat given off by the soda is

$$Q_{\text{soda}} = m_{\text{soda}} c_{\text{W}} (T_{\text{f}} - 20^{\circ}\text{C})$$

- Bring all terms involving Tf on the left-hand-side and all other terms on the right-hand-side.
- Solve for the unknown quantity Tf

$$T_{\text{f}} = \frac{m_{\text{soda}} c_{\text{W}} (20^{\circ}\text{C}) - m_{\text{ice}} L_{\text{f}}}{(m_{\text{soda}} + m_{\text{ice}}) c_{\text{W}}}.$$

Worked Example 4 continued

- The mass of ice is m_{ice} = 3×6.0 g = 0.018 kg and the mass of soda is m_{soda} = 0.25 kg
- Calculate the terms in the numerator:

$$m_{\text{soda}} c_{\text{W}} (20°\text{C}) = (0.25\ \text{kg})(4186\ \text{J/kg}\cdot°\text{C})(20°\text{C}) = 20{,}930\ \text{J}$$

$$m_{\text{ice}} L_{\text{f}} = (0.018\ \text{kg})(334{,}000\ \text{J/kg}) = 6012\ \text{J}$$

- Calculate the denominator:

$$(m_{\text{soda}} + m_{\text{ice}}) c_{\text{W}} = (0.25\ \text{kg} + 0.018\ \text{kg})(4186\ \text{K/(kg}\cdot°\text{C}) = 1122\ \text{J/°C}$$

$$T_{\text{f}} = \frac{20{,}930\ \text{J} - 6012\ \text{J}}{1122\ \text{J/°C}} = 13°\text{C}$$

Note

- This example illustrates the enormous energies involved during a phase change.
- The mass of ice is about 7 percent the mass of water but leads to a noticeable change in the temperature of soda.
- Although we assumed that the ice was at the freezing temperature, this is incorrect: the typical temperature is −6ºC .

Question?

- Why does snow remain on mountain slopes even when daytime temperatures are higher than the freezing temperature?

- Snow is formed from ice crystals and thus is the solid phase of water. Because enormous heat is necessary for phase changes, it takes a certain amount of time for this heat to be accumulated from the air, even if the air is above 0ºC .
- The warmer the air is, the faster this heat exchange occurs and the faster the snow melts.